

Speed and Shape of Dust Acoustic Solitary Waves in the Presence of Dust Streaming

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Dust acoustic solitary waves are investigated on the nonlinear, unmagnetized homogeneous dust ion and dust ion electron plasma with the effects of dust streaming. The Sagdeev's pseudopotential is determined in terms of u_d , the dust ion speed. It is found that there exist a critical value of u_d , beyond which the solitary waves cease to exist.

Key words: Pseudopotential; Solitary Waves; Dusty Plasma.

1. Introduction

There has been a rapidly growing interest in the study of different types of collective processes in dusty plasmas for the last two decades or so. It plays significant roles in space plasma, astrophysical plasma, laboratory plasma and environment. The presence of dusty plasmas in cometary tails, asteroid zones, planetary ring, interstellar medium, lower part of earth's ionosphere and magnetosphere [1 – 7] makes this subject increasingly important. Dusty plasmas also play a vital role in understanding different types of new and interesting aspects in other fields like low temperature physics, radio frequency plasma discharge [8], coating and etching of thin films [9], plasma crystal [10]. Such plasmas are also investigated in laboratory experiments [11 – 13].

Several authors have studied the nonlinear wave phenomena in dusty plasmas. It began with the work of Bliokh and Yarashenko [14] who first theoretically observed the waves in such environment while dealing with waves in Saturn's ring. The discovery of dust-acoustic (DA) waves [15, 16] and dust ion-acoustic (DIA) waves [17, 18] gave a new impetus to the study of waves in dusty plasmas. Later it was found that the dust grain dynamics also introduced few new eigen modes like Dust-Berstein-Greene-Kruskal (DBGK) mode, Dust Lattice (DL) mode [19, 20], Shukla-Verma mode [21], Dust-drift mode [22].

A number of theoretical studies of DIA soliton [23, 24], DA soliton [25, 26] and DL soliton [20] have also been done with low frequency dust electrostatic

and electromagnetic waves. The DIA solitary and shock waves and DL solitary waves were investigated experimentally [6].

To study soliton solution most of the authors however used the reductive perturbative technique (RPT) and obtained Korteweg-de-Vries (KdV) or KdV type equation [27 – 29]. A few years ago Malfliet and Wieers [30] reviewed the studies on solitary waves and found that RPT is based on the smallness of the amplitude. So large amplitude solitary waves were studied by several investigators using Sagdeev's pseudopotential technique [31]. More recently Johnston and Epstein [32] derived Sagdeev's potential in terms of u , the ion-acoustic speed instead of ϕ , the electric potential. They observed that a very small change in the initial conditions destroys the oscillatory behaviour of the solitary waves. Chatterjee and Das [33] also observed the effect of electron inertia on the critical value of u , the ion speed of the solitary waves for which the oscillatory behaviour is destroyed. Maitra and Roychoudhury [34] studied dust acoustic solitary waves by the same technique, considering the dust dynamics in a dusty plasma consisting of warm dust particles and Boltzmann distributed electrons and ions. But they have neglected the effect of dust streaming. There are many situations in space and astrophysical plasmas where the dust streaming effects have to be considered. In planetary magnetosphere and cometary tails two stream instabilities in dusty plasma have been studied [35, 36].

In this paper we consider both the unmagnetized dust ion and dust ion electron plasma. It is found that

the dust streaming has a significant effect on the formation of dust acoustic solitary waves by a similar analysis done in [33, 34].

The organization of the paper is as follows. In Section 2.1. and 2.2. basic equations are written for dust ion and dust ion electron plasma, respectively. The governing second order ODE is derived. A condition for the existence of soliton solution, results and discussion is given in Section 3. Section 4 is kept for conclusions.

2. Basic Equations

Our analysis is based on the fluid's continuity and momentum equations for ions, electrons and the Poisson's equation. For simplicity the discussion is made for the dust ion plasma and the dust ion electron plasma separately. The 1-dimensional case is considered.

2.1. Dust Ion Plasma

The basic equations are

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{z_d}{m_d} \frac{\partial \phi}{\partial x}, \quad (2)$$

$$d_1^2 \frac{\partial^2 \phi}{\partial x^2} = z_d n_d - n_i, \quad (3)$$

where n_d is the dust number density, u_d is the fluid velocity of the dust, z_d is the dust charge, m_d is the dust mass, and ϕ is the electrostatic potential.

We normalize the variables n_d by n_{d0} , n_i by n_{i0} , x by an arbitrary length L , where $d_1 = \frac{\lambda_{di}}{L}$ and $\lambda_{di} = (T_i/4\pi n_{i0} e^2)^{1/2}$ is the ion Debye length. ϕ is normalized by $\frac{T_i}{e}$. u_d, u_{d0} are normalized by $\beta_1 c_{sd}$, where $\beta_1^2 = z_d$ and $c_{sd} = (T_i/m_d)^{1/2}$ is the speed of the dust acoustic wave. Time t is normalized by ω_{pd}^{-1} , where $\omega_{pd} = (4\pi n_{d0} z_d^2 e^2/m_d)^{1/2}$. (For details see [37].)

In order to search for solitary waves which solves (1) to (3), we introduce a linear substitution $\xi = x - vt$

admitting only solutions which depend in space and time in the form of the wavy variable $x - vt$. By substituting $\frac{\partial}{\partial x} = \frac{d}{d\xi}$ and $\frac{\partial}{\partial t} = -v \frac{d}{d\xi}$ equations (1)–(3) reduce to

$$-v \frac{dn_d}{d\xi} + \frac{d(n_d u_d)}{d\xi} = 0, \quad (4)$$

$$-v \frac{du_d}{d\xi} + u_d \frac{du_d}{d\xi} = \frac{z_d}{\beta_1^2} \frac{d\phi}{d\xi}, \quad (5)$$

$$d_1^2 \frac{d^2 \phi}{d\xi^2} = z_d n_d - n_i. \quad (6)$$

The boundary conditions are: $\phi, \frac{d\phi}{d\xi} \rightarrow 0, n_d \rightarrow 1, u_d \rightarrow u_{d0}, m_d \rightarrow \beta_1^2$, as $|\xi| \rightarrow \infty$.

Integrating (4) and using the boundary conditions given above we get

$$n_d = \frac{v - u_{d0}}{v - u_d}. \quad (7)$$

From (5) we obtain

$$v - u_d = [(v - u_{d0})^2 + \frac{2z_d \phi}{\beta_1^2}]^{1/2}. \quad (8)$$

Therefore

$$n_d = \frac{v - u_{d0}}{[(v - u_{d0})^2 + \frac{2z_d \phi}{\beta_1^2}]^{1/2}} \quad (9)$$

and

$$n_i = e^{-\phi}. \quad (10)$$

Let $\delta = \frac{n_{d0}}{n_{i0}}$, now using (9), (10) in (6), we find

$$\frac{d^2 u_d}{d\xi^2} = \frac{d\psi_d}{du_d}, \quad (11)$$

where

$$\psi_d = -\frac{\psi}{(\frac{d\phi}{du_d})^2} \quad (12)$$

and

$$\psi = (1 - e^{-\psi_1}) + \delta \beta_1^2 (v - u_{d0})^2 \left[1 - \left(1 + \frac{2z_d \psi_1}{\beta_1^2 (v - u_{d0})^2} \right)^{1/2} \right] \frac{1}{d_1^2}, \quad (13)$$

where

$$\psi_1 = \frac{\beta_1^2}{2z_d} [(v - u_d)^2 - (v - u_{d0})^2]. \quad (14)$$

Thus

$$\frac{d^2 u_d}{d\xi^2} = -\frac{z_d}{d_1^2 \beta_1^2 (v - u_d)} \left[\frac{\delta z_d}{\left(1 + \frac{2z_d v_1}{\beta_1^2 (v - u_{d0})^2}\right)^{1/2}} - e^{-v_1} \right] + \frac{2z_d^2}{d_1^2 \beta_1^2 (u_d - v)^3} \psi. \quad (15)$$

2.2. Dust Ion Electron Plasma

In this case the basic equations are

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \quad (16)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{z_d}{m_d} \frac{\partial \phi}{\partial x}, \quad (17)$$

$$d_2^2 \frac{\partial^2 \phi}{\partial x^2} = z_d n_d + n_e - n_i, \quad (18)$$

where n_α ($\alpha = i, e, d$) is the number density of the species, u_d is the fluid velocity of the dust, z_d is the dust charge, m_d is the dust mass, and ϕ is the electrostatic potential.

We normalize the variables n_d by n_{d0} , n_i by n_{i0} , n_e by n_{e0} , x by an arbitrary length L where $d_2 = \frac{\lambda_{De}}{L}$ and $\lambda_{De} = (T_e/4\pi n_{e0} e^2)^{1/2}$ is the ion Debye length. ϕ is normalized by T_e/e . u_d, u_{d0} are normalized by $\beta_2 c_{sd}$, where $\beta_2^2 = Z_d(d-1)/(d+1)$, where $d = n_{i0}/n_{e0}$ and $c_{sd} = (T_e/m_d)^{1/2}$ is the speed of the dust acoustic wave. Time t is normalized by ω_{pd}^{-1} , where $\omega_{pd} = (4\pi n_{d0} Z_d^2 e^2/m_d)^{1/2}$. (For details see [37].)

In equilibrium we have

$$n_{i0} = n_{e0} + z_d n_{d0}. \quad (19)$$

In order to search for solitary waves for (16) to (18), we introduce again the variable $\xi = x - vt$. Substituting $\partial u/\partial x = du/d\xi$ and $\partial u/\partial t = -v du/d\xi$ equations (16)–(18) reduce to

$$-v \frac{dn_d}{d\xi} + \frac{d(n_d u_d)}{d\xi} = 0, \quad (20)$$

$$-v \frac{du_d}{d\xi} + u_d \frac{du_d}{d\xi} = \frac{z_d}{\beta_1^2} \frac{d\phi}{d\xi}, \quad (21)$$

$$d_2^2 \frac{d^2 \phi}{d\xi^2} = z_d n_d + n_e - n_i. \quad (22)$$

The boundary conditions are: $\phi, d\phi/d\xi \rightarrow 0$, $n_d \rightarrow 1$, $u_d \rightarrow u_{d0}$, $m_d \rightarrow \beta_2^2$, as $|\xi| \rightarrow \infty$.

Integrating (20) and using the boundary conditions given above we get

$$n_d = \frac{v - u_{d0}}{v - u_d}. \quad (23)$$

Again from (21) we find

$$v - u_d = \left[(v - u_{d0})^2 + \frac{2z_d \phi}{\beta_2^2} \right]^{1/2}. \quad (24)$$

Therefore

$$n_d = \frac{v - u_{d0}}{\left[(v - u_{d0})^2 + \frac{2z_d \phi}{\beta_2^2} \right]^{1/2}} \quad (25)$$

and

$$n_i = e^{-\sigma \phi}, \quad (26)$$

where $\sigma = T_e/T_i$ and

$$n_e = e^\phi. \quad (27)$$

Let $d = n_{i0}/n_{e0}$, using (25), (26), (27) in (22), we get as in the previous subsection

$$\frac{d^2 u_d}{d\xi^2} = -\frac{z_d}{d_2^2 \beta_2^2 (v - u_d)} \left[\frac{d-1}{\left(1 + \frac{2z_d v_1}{\beta_2^2 (v - u_{d0})^2}\right)^{1/2}} + e^{v_1} - d e^{-\sigma v_1} \right] + \frac{2z_d^2}{d_2^2 \beta_2^2 (u_d - v)^3} \psi. \quad (28)$$

where

$$\psi = (1 - e^{v_1}) + \frac{(d-1)}{z_d} \beta_2^2 (v - u_{d0})^2 \left[1 - \left(1 + \frac{2z_d v_1}{\beta_2^2 (v - u_{d0})^2}\right)^{1/2} \right] + \frac{d}{\sigma} (1 - e^{-\sigma v_1}) \frac{1}{d_2^2}. \quad (29)$$

All other variables are the same as in the previous subsection.

3. Soliton Solution, Results and Discussion

To find the region of existence of solitary waves, one has to study the nature of the function $\psi_d(u_d)$ and $\phi_1(u_d)$ defined by

$$\psi_d(u_d) = \frac{(u'_d)^2}{2}, \quad (30)$$

where

$$u''_d = \frac{\partial \psi_d}{\partial u_d} = \phi_1(u_d). \quad (31)$$

For solitary waves (see [32–34]) $\phi_1(u_d)$ will have two real roots, one being at $u_d = 0$ and the other at some point $u_d = u_{d1} (\geq 0)$. Also $\phi_1(u_d)$ should be positive on the interval $(0, u_{d1})$ and negative on $(u_{d1}, u_{d\max})$, where $u_{d\max}$ is obtained from the nonzero root of $\psi_d(u_d)$. To get the shape of the travelling solitary wave one has to solve $\phi_1(u_d) = u''_d$ numerically with suitable boundary conditions.

3.1. Dust Ion Plasma

Figure 1 shows the plot of $\psi_d(u_d)$ vs u_d for $\nu = 1.1$. The other parameters are $u_{d0} = 0, z_d = 100, \beta_1^2 = 100, \delta = 0.01$. One can see that $\psi_d(u_d)$ crosses the u_d axis at $u_d = u_{dc} = 0.293118$. Hence the amplitude of the solitary wave for this set of parameters will be 0.293118. Obviously $u_d = u_{dc}$ is the critical value of u_d at which a singularity occurs. It can be seen from (8) and (9) that u_{dc} is the value of u_d beyond which n_d , the dust density, becomes complex.

To get the shape of the solitary wave we have solved numerically $u''_d = \phi_1(u_d)$ with $u_{dc} = 0.293118, u'_d = 0$ and Fig. 2a depicts the soliton solution $u_d(\xi)$ plotted against ξ . The other parameters are the same as those

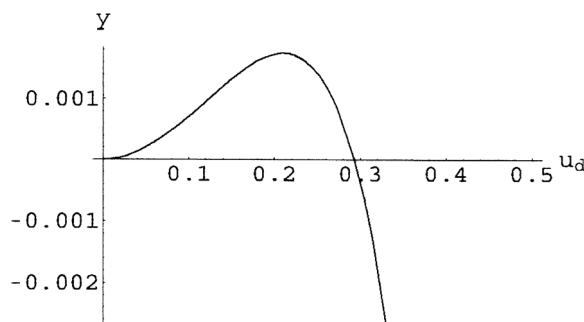


Fig. 1. Plot of $\psi_d(u_d)$ vs u_d for $\nu = 1.1$. The other parameters are $u_{d0} = 0, z_d = 100, \beta_1^2 = 100, \delta = 0.01, d_1 = 1$.

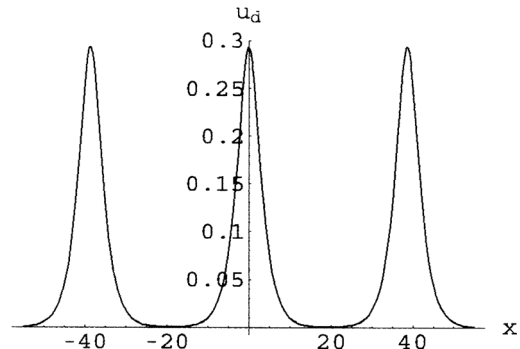


Fig. 2a. The soliton solution $u_d(\xi)$ plotted against ξ for $u_{dc} = 0.293118$. The other parameters are same as those in Figure 1.

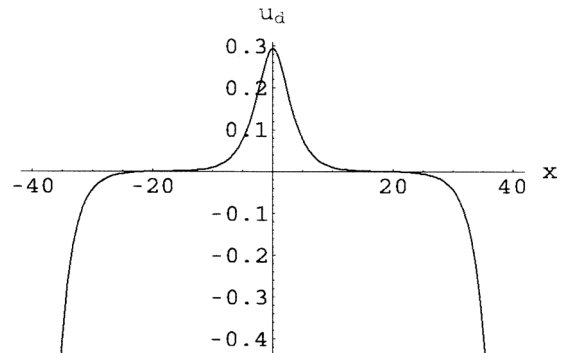


Fig. 2b. The soliton solution $u_d(\xi)$ plotted against ξ for $u_{dc} = 0.293119$. The other parameters are same as those in Figure 1.

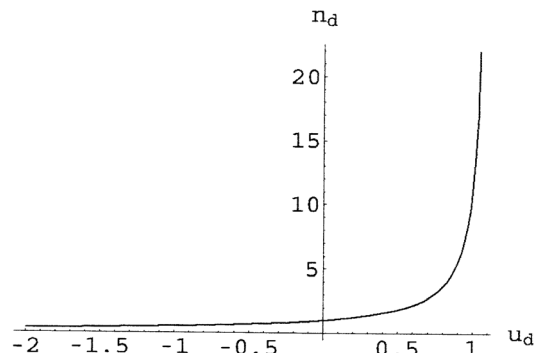


Fig. 3a. Plot of n_d vs u_d for $\nu = 1.1$ and $u_{d0} = 0.05$.

in Figure 1. For $u_d > u_{dc}$ the soliton solution cease to exist and it is shown in Figure 2b. In this figure u_{dc} is taken as 0.293119 (all the other parameters are same as in Figure 1). Hence it is seen that a small change of the value of u_{dc} can destroy the periodic behaviour of the solitary wave.

To see the role of compressibility, n_d is plotted against u_d in Figure 3a. Parameters are same as those in

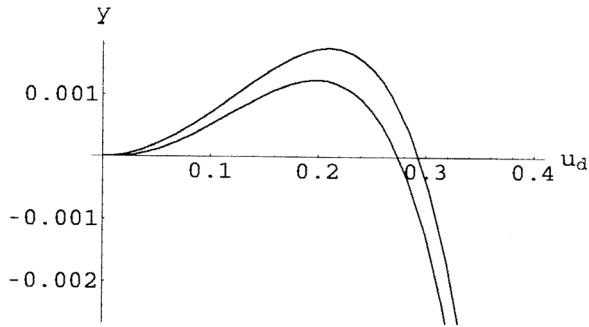


Fig. 3b. Plot of $\psi_d(u_d)$ vs u_d for $u_{d0} = 0$ and 0.05. The other parameters are same as those in Figure 1.

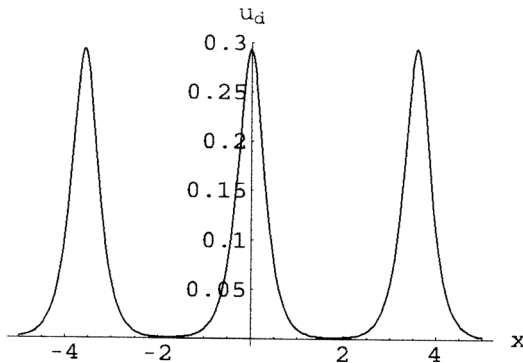


Fig. 3c. The soliton solution $u_d(\xi)$ plotted against ξ for $u_{dc} = 0.293118$ and $d_1 = 0.1$. The other parameters are same as those in Figure 1.

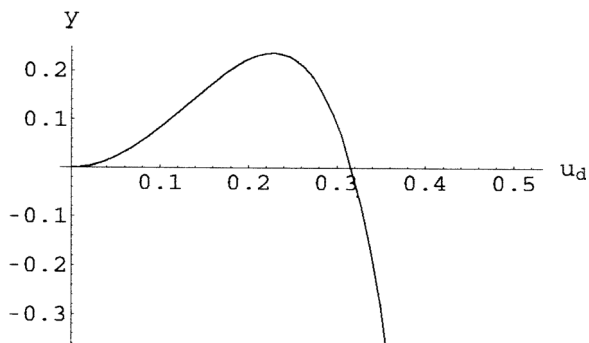


Fig. 4. Plot of $\psi_d(u_d)$ vs u_d for $\nu = 1.1$. The other parameters are $u_{d0} = 0$, $z_d = 100$, $\beta_2^2 = 100$, $\delta = 0.01$, $d = 101$, $\sigma = 1$, $d_2 = 1$.

Figure 1. For $n_d > 1$ waves are called compressive and for $n_d < 1$ those are called rarefractive. In Fig. 3b ψ_d is plotted against u_d in the absence ($u_{d0} = 0$) as well as in the presence ($u_{d0} = 0.05$) of dust streaming effect. Other parameters are same as those in Figure 1. Here it is seen that the amplitude of the solitary wave

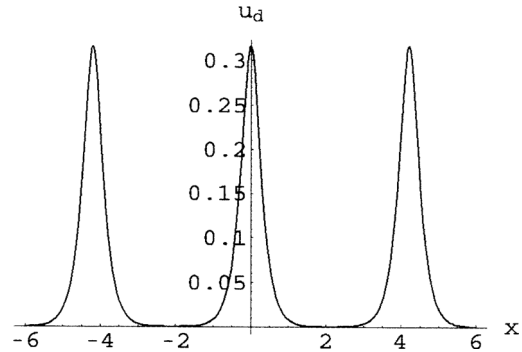


Fig. 5a. The soliton solution $u_d(\xi)$ plotted against ξ for $u_{dc} = 0.3163202$. The other parameters are same as those in Figure 4.

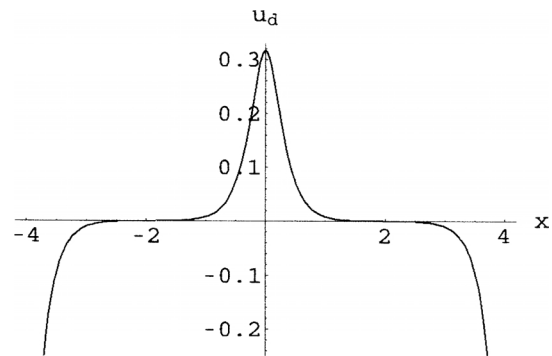


Fig. 5b. The soliton solution $u_d(\xi)$ plotted against ξ for $u_{dc} = 0.3163203$. The other parameters are same as those in Figure 4.

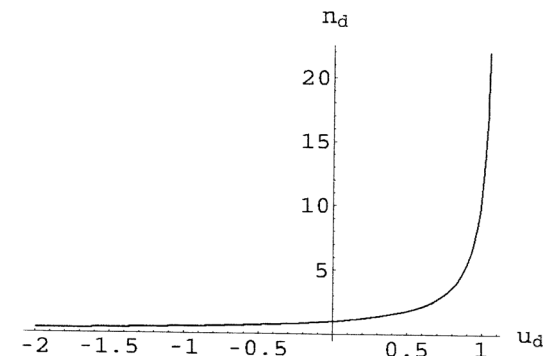


Fig. 6a. Plot of n_d vs u_d for $\nu = 1.1$ and $u_{d0} = 0.05$.

decreases significantly in the presence of dust streaming effect. The effect of dimensionality is seen in Figure 3c. In this figure u_d is plotted against ξ for $d_1 = 0.1$. Comparing Fig. 2a and Fig. 3c one can see that the amplitude of the solitary waves are same in both the cases but the width of the solitary wave decreases with the decrease of d_1 .

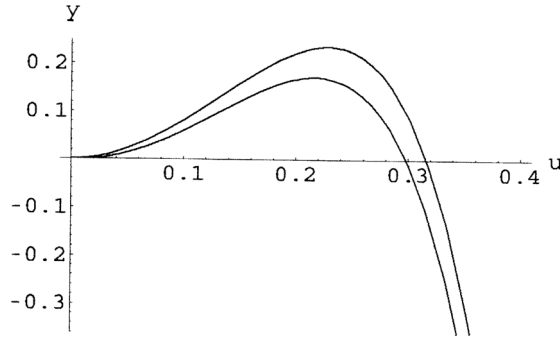


Fig. 6b. Plot of $\psi_d(u_d)$ vs u_d for $u_{d0} = 0$ and 0.05 . The other parameters are same as those in Figure 4.

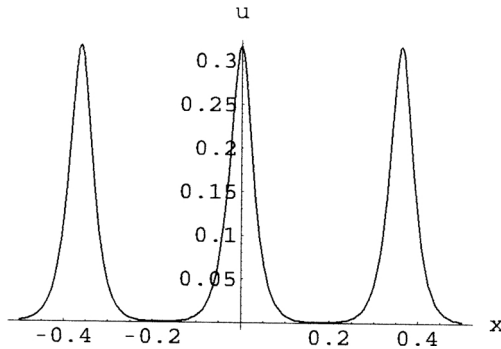


Fig. 6c. The soliton solution $u_d(\xi)$ plotted against ξ for $u_{dc} = 0.3162202$ and $d_2 = 0.1$. The other parameters are same as those in Figure 4.

3.2. Dust Ion Electron Plasma

Figure 4 shows the plot of $\psi_d(u_d)$ vs u_d for $v = 1.1$. The other parameters are $u_{d0} = 0, z_d = 100, \beta_2^2 = 100, \delta = 0.01, d = 101, \sigma = 1$. It is seen that the function $\psi_d(u_d)$ crosses the u_d axis at $u_d = u_{dc} = 0.3163202$. Hence the amplitude of the solitary wave for this set of parameters will be 0.3163202 . To get the shape of the solitary wave we have solved numerically $u_d'' = \phi_1(u_d)$ with $u_{dc} = 0.3163202, u_d' = 0$ and Fig. 5a depicts the soliton solution $u_d(\xi)$ plotted against ξ . The other parameters are same as those in Figure 4. It is seen that $u_{dc} = 0.3163202$ is the critical value for u_d . For $u_d > u_{dc}$ the soliton solution ceases to exist and

it is shown in Figure 5b. In this figure u_{dc} is taken as 0.3163203 (all the other parameters are the same as those in Fig. 4). Hence it is seen that a small change of the value of u_d can destroy the periodic behaviour of the solitary wave.

To see the role of compressibility, n_d is plotted against u_d in Figure 6a. Parameters are same as those in Figure 4. For $n_d > 1$ waves are called compressive and for $n_d < 1$ those are called rarefractive. In Fig. 6b ψ_d is plotted against u_d in the absence ($u_{d0} = 0$) as well as in the presence ($u_{d0} = 0.05$) of dust streaming effect. Other parameters are same as those in Figure 4. Here it is seen that the amplitude of the solitary wave decreases significantly in the presence of dust streaming effect. The effect of dimensionality is seen in Figure 6c. In this figure u_d is plotted against ξ for $d_2 = 0.1$. Comparing Figs. 5a and 6c one can see that the amplitude of the solitary waves are same in both the cases but the width of the solitary wave increases with the increase of d_2 .

4. Conclusion

Using the pseudopotential approach we have studied the speed and shape of the dust acoustic solitary waves in homogeneous unmagnetized plasma with the presence of dust streaming. Both cases, dust ion and dust ion electron plasmas, are considered. Sagdeev's potential is obtained in terms of u_d , the dust fluid velocity. It is seen that there exists a critical value of u_d , at which $u_d'^2 = 0$, beyond which the soliton solution does not exist. This technique can be extended to the study of the existence of the dust acoustic solitary waves for vortex like ion distribution, non-thermal distribution of electrons etc. Work in this direction is in progress.

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